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Name of Principal Author and all other author(s): Thomas F. Chwastyk

Principal Author's Organization and address:

AFSAA/SAAT
1777 N Kent St
Rosslyn VA 22209

Phone: DSN 425-8672 (Com 703-588-8672)
Fax: DSN 425-8738 (Com 703-588-8738)
Email: Thomas.Chwastyk@pentagon.af.mil

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Headquarters U.S. Air Force

Integrity - Service - Excellence

Design of Experiments (DOE) for Validation of STORM Based on THUNDER

73rd MORSS, USMA 2005
WG 25 & 29

Mr. Tom Chwastyk
AFSAA/SAAT

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STORM

Purpose

- THUNDER vs. STORM
- Goal of DOE here: V&V by comprehensive comparison
  - Why not VV&A?
- General and specific models of the V&V experiment
- DOE for adequate excitation to compare responses
  - Screening to reduce response model size
  - Adding design points to check for interactions
- Comparing THUNDER and STORM response
  - By Singular Value Decomposition
  - By Canonical Correlation
- Automating execution of runs and extraction of results
  - Work in progress – all feedback welcome
- Punch line: Massive quantitative validation experiment – uses computers and sophisticated techniques to exceed past human limits on data production and interpretation

THUNDER vs. STORM

- THUNDER: accepted standard
  - Not Government owned
  - Proprietary engine, SIMSCRIPT
    - CACI licenses – big $$$ / seat / year
  - Ground war:
    - "Pistons" define FLOT
    - Piston walls do not reorient as FLOT distorts
  - No study manager -- manual file configuration control
  - Minimal GUI
  - Minimal ISR

- STORM: AFSAA's next standard
  - Government owned
  - Open source, non-proprietary
    - Simulation engine in g++ ; maps, DB tools; even OS, all open
    - No per-seat license fees
  - Ground war:
    - Along arcs and within nodes
    - Shape of FLOT morphs naturally under network constraints
  - Study manager automates file configuration control
  - Extensive GUI – map tool, report tool, graph tool
  - Sat tool and other significant ISR developments

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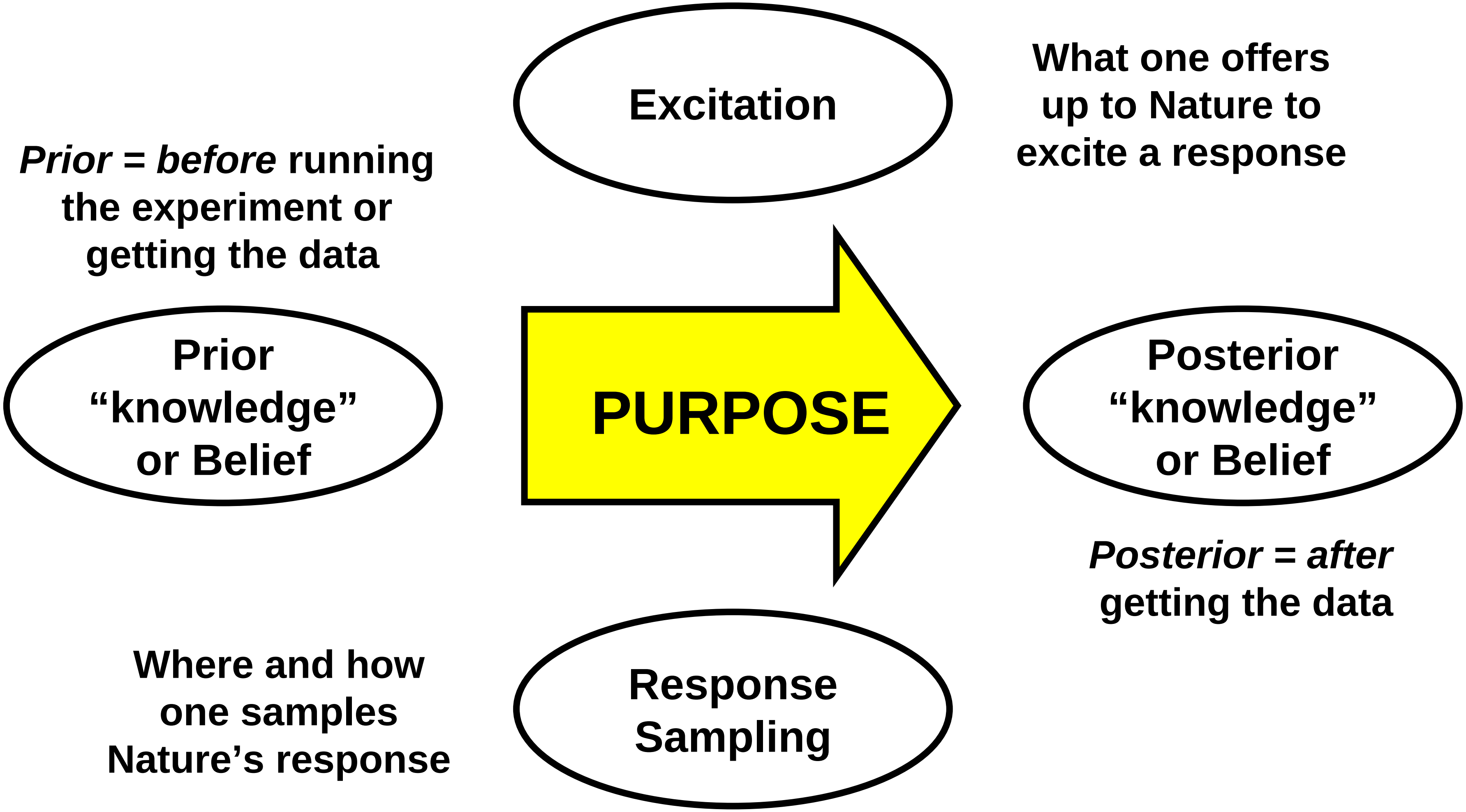
V&V by Comparison (Not VV&A?)

- V&V
  - Verification: "Solve the Problem Right"
    - Identify and fix lurking code problems
  - Validation: "Solve the Right Problem"
    - Accepted referent is THUNDER but comparison is a hard problem
  - Comparison needed <u>over a set of scenarios of interest</u>
    - Are results "comparable to or better than" THUNDER? Just what does that mean?
      - SME's "face validation" vs. statistical tools – both are needed
    - Does DOE show effects of changes in STORM vs. THUNDER?
- Why V&V, not VV&A?
  - A (Accreditation) is PM's decision – balancing risk versus resources
    - Too little V&V: low resource expenditure but high risk
    - Too much V&V: little risk reduction for last expenditures

A General Model of an Experiment



…Tailored to V&V and Comparison

STORM uses next generation THUNDER campaign model

Excitation

Excitation is the set of scenarios, each with its experimental design and with levels for factors: <u>the INPUTS</u>

Prior knowledge or Belief

PURPOSE

Posterior knowledge or Belief

Response Sampling is MOMs + dofs extracted from simulation: <u>the OUTPUTS</u>

Response Sampling

STORM improves on THUNDER – it's a good replacement

V&V Scenario Set of Interest

- Consult with SMEs, see for what campaign models/scenarios THUNDER results (inputs and outputs) are already available and of interest
  - THUNDER has been much more expensive to run (small error $\Rightarrow$ typically 60-100 reps) than STORM – hence, minimize additional THUNDER runs
- Choose among available THUNDER campaigns to agree on sufficiently complete V&V scenario set
- If necessary, add THUNDER or STORM work to achieve common and complete set of scenarios of significant interest for V&V

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Need to Reduce Input Dimensions

THUNDER	v6.9	99	*.dat files
STORM	v1.3	115	*.dat files (plus path.dat)
- Typical .dat file has 10 to 1000+ parameters (inputs) – highly interrelated, often nonlinearly
- Too many parameters to screen individually
  - Use SME's to combine (cull?) inputs into fewer (10 to 100) factors $f_i \equiv_{def} [\ p_{i,j'}\ ]$ parameter vectors
  - $p_{i,j}$, $p_{i,k\neq j}$ need not be in same .dat file
  - Each parameter in a factor has two levels, A / B – goes A / B as its factor goes high / low – no "within-factor interactions"
- Factor screening takes place in STORM across entire V&V scenario set at once
- Whether the change in level of factor f_i is significant depends both on the scenario set and on what MOM(s) or output dof(s) are of interest

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Input Sensitivity Screening

- **Initial screening in STORM of two-level factors**
  - **Resolution V (preferred) or IV (if necessary) fractional factorial (FF) – possibly even res. III for initial stab?**
    - **If resolution IV necessary, follow up with resolution V (res. IV significant factors only, plus all 2-factor interactions) to screen for significant interactions**
  - **256 reps (resolution IV) $\Rightarrow$ up to 128 factors**
    - **Comparable to # of reps in STORM V&V reports for previous releases (19 + cases x 10 repeated reps / case)**
    - **Likely fewer factors ($\leq$64, $\leq$32) if long reps (t_{sim}>>10 days)**
  - ***However*, 256 reps (resolution V) $\Rightarrow$ only 12 to 17 factors (+ all 2-level interactions)**
- **In any case –**
  - **NOT One-Factor-At-a-Time as before!**
  - **NO replicated design points (reps)!**

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Matrix Model Equations

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E}$$

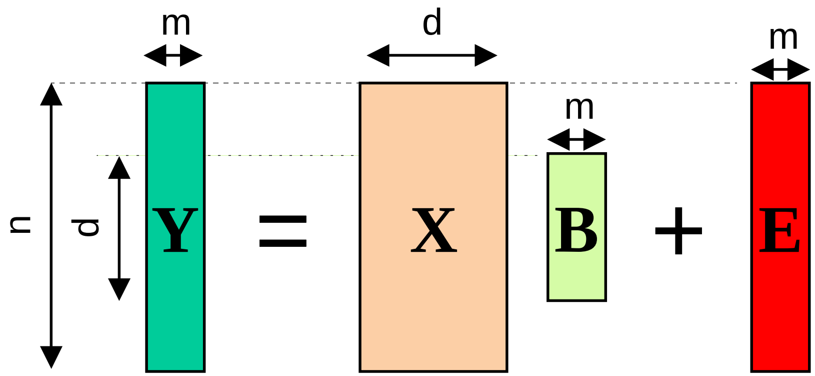
$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\mathbf{B}} + \hat{\mathbf{E}}(\mathbf{s})$$

$$\hat{\mathbf{E}}'_i = \left[f_1(s_i) \ \cdots \ f_j(s_i) \ \cdots \ f_m(s_i) \right]'$$

$\hat{\mathbf{B}}$ is by least squares, minimizes

$$rss = \sum_i \sum_j (y_{ij} - \hat{y}_{ij})^2$$

Not Your Usual Few Responses



m = # of response variables

d = # of design variables (factors)

n = # of runs

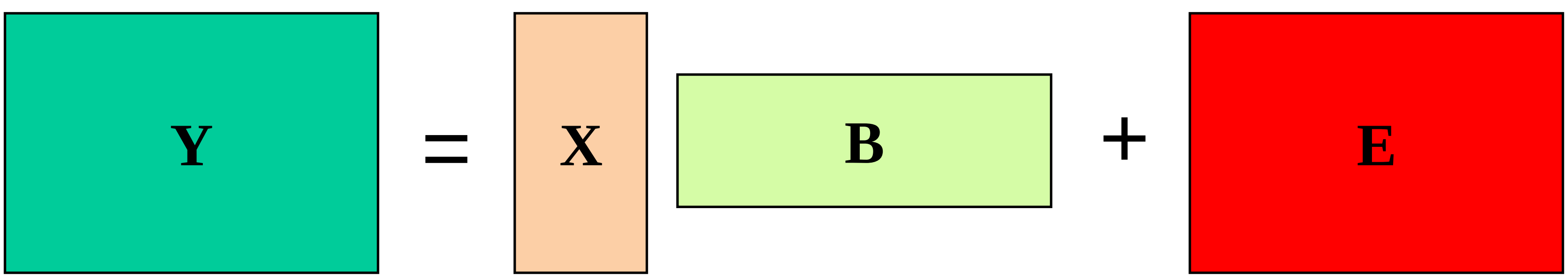
n > d (always)

m << d (usually)

m >> d

Why does Y need to be so broad, m >> d ?

In our V&V experiment, X is <u>excitation</u>; Y samples <u>response</u>. We are deliberately oversampling.



We oversample by several times the dimensionality of our excitation in a “shotgun approach”, hoping we catch most of the significant dimensions of model response.

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Why Fit Is Always Too Good

$$\mathbf{Y} = \mathbf{X}\,\mathbf{B} + \mathbf{E}$$

$$\hat{\mathbf{Y}} = \mathbf{X}\,\hat{\mathbf{B}} + \hat{\mathbf{E}}$$

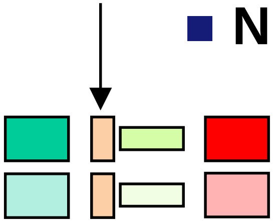
$$\mathbf{X}'\,\mathbf{E} \neq 0 = \mathbf{X}'\,\hat{\mathbf{E}}$$

Elements of Matrix Model

- Starts as typical multivariate response model
  - Response matrix $\mathbf{Y}$ (n×m)
  - Design matrix $\mathbf{X}$ (n×d)
  - Effects matrix $\mathbf{B}$ (d×m)
  - "Out-of-model" matrix $\mathbf{E}$ (n×m)
  - Estimates "hatted": $\hat{\mathbf{E}}$, $\hat{\mathbf{B}}$
- Beyond typical model:
  - *Compute* $\mathbf{Y}$ from $\mathbf{X}$ in permuted order $pco^{(1)}$, $pco^{(2)}$, $pco^{(3)}$, but *record* $\mathbf{Y}$ in design order (picture of $\mathbf{Y}$, pco_i, $pco^{(i)}$ later)
  - Record random seed and permuted computational order $[\, s_i \;\; pco_i \,]$ for row i (nx2)

Design Matrix X - Words

- ROWS in the design matrix X
  - Give particular combination of factor levels for each design point
  - Cover all scenarios of interest
  - Contain a non-zero entry in only one of the columns indicating scenarios
- COLUMNS in the design matrix X
  - Signify design (input) factors
  - Signify scenarios for estimating mean
    - Decompose sum of squares due to scenario means
    - Number of means is included in d, # of design variables



Design Matrix - Picture

X

scenario # 1	2	3	4	5	6	7	8	9	10 factor level
1	0	0	0	1	3	5	7	2	8
1	0	0	0	2	4	6	7	2	8
1	0	0	0	2	4	5	7	2	8
1	0	0	0	1	3	6	7	2	8
0	1	0	0	1	3	5	8	7	1
0	1	0	0	2	4	6	8	7	1
0	1	0	0	2	4	5	8	7	1
0	1	0	0	1	3	6	8	7	1
0	0	1	0	1	3	6	8	7	8
0	0	1	0	1	3	5	8	2	1
0	0	1	0	1	3	6	7	7	8
0	0	1	0	1	4	5	7	2	1
0	0	1	0	1	4	6	8	7	8
0	0	1	0	1	4	5	7	2	1
0	0	0	1	1	4	6	8	2	8
0	0	0	1	2	3	6	7	2	8
0	0	0	1	1	3	6	8	2	1
0	0	0	1	2	3	5	7	7	1
0	0	0	1	1	3	5	7	7	8
0	0	0	1	2	4	5	8	7	8

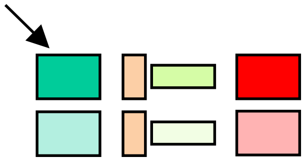
1's account for scenario means, ssq

constant factor level within scenario says significance of factor is not tested within scenario

Notional design only, not 2 $^{k-p}$

Response Matrix Y - Words

- ROWS in the response matrix Y
  - Correspond to design matrix rows
- COLUMNS in the response matrix Y signify
  - Scalar or vector MOMs / dofs
  - Consistent responses for each row and scenario
    - Squadrons or other entity types handled differently by planning files on different design points should not re-use the same response columns unless
      - the difference in response due to difference in planning file parameters is the effect being sought and
      - the difference in planning file parameters is reflected in one or more factors



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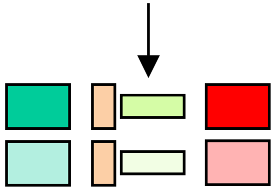
Response Matrix - Picture

$pco^{(i)}$	pco_i	scenario #	i = row # in X	Scalar MOM 1	Scalar MOM 2	Scalar MOM 3	Vector dof 1 day 1	Vector dof 1 day 2	Vector dof 1 day 3	…	Vector dof 1 day n-1	Vector dof 1 day n	Scalar MOM 4	Vector dof 2 day 1	Vector dof 2 day 2	Vector dof 2 day 3	…	Vector dof 2 day n-1	Vector dof 2 day n	Scalar MOM 5	Vector dof 3 day 1	Vector dof 3 day 2	…	Vector dof 3 day n-1	Vector dof 3 day n	Vector dof 4 day 1	… etc.
13	14	1	1																								
11	3	1	2																								
2	5	1	3																								
7	9	1	4																								
3	11	2	5																								
10	12	2	6																								
12	4	2	7																								
8	8	2	8																								
4	13	3	9																								
14	6	3	10																								
5	2	3	11																								
6	7	3	12																								
9	1	3	13																								
1	10	4	14																								
…	…	…	…																								

$\mathbf{Y}$

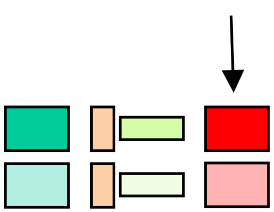
Effects Matrix B, B hat

- ROWS in the effects matrix B
  - Correspond to design variables
  - For SME-aggregated factors, coefficient shows effect of change in level
  - For scenario indicator variables, coefficient shows scenario mean for each response
- COLUMNS in the effects matrix B
  - Correspond to response MOMs, dofs



Out-of-Model Matrix E, E hat

- **Part of E is assumed due to stochastic effects**
  - **Neither random seed value s_i nor its effect $f_j(s_i)$ on each response y_{ij} is known to the fitting process**
  - **s_i and $f_j(\bullet)$ — hence $f_j(s_i)$ — are termed exogenous (excluded from model)**
- **Effects of exogenous variables, *including stochastic effects*, alias to an unknown extent onto endogenous (included in model) effects b_j**



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Interactions, Response Surface

- Once factors and 2-factor interactions are identified as significant at resolution V, high-low level only …
- Add intermediate levels as appropriate to gain Response Surface Model (RSM)
  - Central composite design if appropriate
- Retain only <u>statistically</u> significant factors (and interactions) for comparing STORM to THUNDER
  - Not normal statistical practice to "cherry pick" interactions without physical model justification
  - Empirically justifiable because RSM is <u>descriptive</u> only, not first principles
  - We are a long way off from this

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Dimension Reduction (1): RSM

- **Add face and center points for significant factors to get central composite design if feasible**
- **Possibly use intermediate points for near-orthonormal Latin Square**
- **Get response surface model (RSM) – polynomial or spline**
  - **Stop on Mallow's C_p or other measure of complexity vs. goodness of fit**
- **<u>Retains coordinates of original model space</u>**

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Weight Questions (1)

- **Question 1: What is the proper weight or scale per MOM / dof(s) in the penalty function for lack of fit?**
  - **That is, how much should lack of fit in one column (or set of columns) of Y count relative to lack of fit in another column (or set of columns)?**
- **Question 2: Same as Question 1 but for one scenario (set of rows) versus another**

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Weight Answers (Partial) (2)

- Depends on user goal and on STORM / THUNDER comparison approach
  - Data for SVD, canonical correlation should have zero scenario mean
  - Appears no natural scale for Singular Value Decomposition (SVD); use unit column variance for consistency with canonical correlation
  - Scale set to unit column variance by definition of canonical correlation, but meaning and appropriateness to user goal are not clear
  - Beware augury / mysticism / wise chin-scratching

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Equations in Passing

- Final dry run for this brief commented that there were way too many equations, too few graphics
- Graphics proved far more time consuming to produce than initially expected
- Equations were retained more as art objects and speaking totems than as detailed communication tools
- Equations past the basic “Matrix Model Equations” will not be on the midterm or the final

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Dim. Red. (2): SVD (1)

- **Singular Value Decomposition (SVD) on responses Y adjusted to zero-scenario-mean, scaled to unit Y column norm, and transposed*:**

* not a good idea, but not fatal

$$\underset{(n\times m)}{\mathbf{Y}_{zsm}} = \mathbf{Y}(\mathbf{I}_n - \underbrace{\mathbf{X}_{sm}(\mathbf{X}_{sm}'\mathbf{X}_{sm})^{-1}\mathbf{X}_{sm}'}_{\mathbf{X}_{sm}=\text{only columns for scenario means}})$$

$$\underset{(m\times n)}{\mathbf{A}} = \left(\hat{\mathbf{Y}}_{zsm}\right)', \left(\hat{y}_{zsm}\right)_{ij} = \left(y_{zsm}\right)_{ij} \Big/ \sqrt{\sum_{i'=1}^{n}\left(y_{zsm}\right)_{i'j}^2}$$

$$\mathbf{A} = \mathbf{ODV}' \begin{cases} \mathbf{O}'\mathbf{O} = \mathbf{I}_n, (m > n) \Rightarrow \mathbf{OO}' \neq \mathbf{I}_m \\ \mathbf{D} \text{ diagonal, p.s.d.}, (i < j) \Rightarrow (d_{ii} \geq d_{jj}) \\ \mathbf{VV}' = \mathbf{V}'\mathbf{V} = \mathbf{I}_n \end{cases}$$

… SVD (2)

- **Continuing,**

$$\mathbf{A}'\mathbf{A} = \mathbf{Z} = \mathbf{V}\mathbf{D}'\mathbf{O}'\mathbf{O}\mathbf{D}\mathbf{V}'$$
$$= \mathbf{V}\mathbf{D}^2\mathbf{V}'$$
$$\mathbf{Z}\mathbf{V} = \mathbf{V}\mathbf{D}^2\mathbf{V}'\mathbf{V} = \mathbf{V}\mathbf{D}^2$$
$$\mathbf{Z}\mathbf{V} = \mathbf{V}\boldsymbol{\Lambda} \quad \text{(symmetric eigenproblem)}$$
$$d_{ii} = \sqrt{\lambda_i} \quad (\text{sorted } \lambda_1 \geq \lambda_2 \geq \ldots \lambda_n \geq 0)$$
$$\mathbf{O}_i = \mathbf{A}\mathbf{V}_i d_{ii}^{-1} \quad \text{canonical column } i \text{ of } \mathbf{A} \text{ for } d_{ii} > 0$$

- **Why is this significant?**

… SVD (3)

- This is significant *because*…

$$\mathbf{A} = \mathbf{ODV'} = \sum d_{ii} \mathbf{OV'} \text{ (}\underline{\text{exactly!}}\text{) and}$$

$$\sum_j \sum_k (\mathbf{O}_i \mathbf{V}_i')^2_{jk} = \sum_j \sum_k (o_{ji} v_{ik})^2 = \sum_j o^2_{ji} \sum_k v^2_{ik} = 1,$$

so <u>all scale information is in the "singular values"</u> d_{ii} in the sense that d_{ii}^2 shows how many unit rows of A are expressed by $d_{ii}\mathbf{V}'_i$.

- This means that the completeness $R^2(k,m)$ of expressing all m unit rows of the response matrix A using only k coefficients is

$$R^2_{k,m} = \left(\sum_{i=1}^{k} d^2_{ii}\right) \Big/ \left(\sum_{i'=1}^{m} d^2_{i'i'}\right)$$

- More on SVD follows after discussion on Canonical Correlation

DR (3): Canonical Correlation (1)

- Two response sets, say $\mathrm{T}=\hat{\mathrm{Y}}'_{\mathrm{T}}$ for THUNDER and $\mathrm{S}=\hat{\mathrm{Y}}'_{\mathrm{S}}$ for STORM
- *m* ROWS of T, S
  - Zero scenario mean, unit variance as for SVD
  - Same row ⇔ same MOM or dof for T as for S
- *n* COLUMNS of T, S are in same order as design matrix rows, although design points were computed in permuted sequence(s)

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… CanCorr (2)

- **Canonical correlation (CanCorr) gives** $m\times 2$ **coefficients** $[\,\mathbf{lca}_k\ \mathbf{lcb}_k\,]$ **for successive** $n\times 2$ **linear composites** $[\,\mathbf{u}_k\ \mathbf{w}_k\,]$**, such that for** $\mathbf{u}_k' = \mathbf{lca}_k'\mathbf{T}$, $\mathbf{w}_k' = \mathbf{lcb}_k'\mathbf{S}$, $k = 1, 2, \ldots, \min(m,n)$
  - $\mathbf{u}_k$ **and** $\mathbf{w}_k$ **have maximal correlation** ρ^*_k **between any two linear composites of** T, S **<u>not</u> spanned by** $[\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_{k\text{-}1}]'$, $[\mathbf{w}_1\ \mathbf{w}_2\ \ldots\ \mathbf{w}_{k\text{-}1}]'$
  - $\mathbf{Var}(\mathbf{u}_k) = \mathbf{Var}(\mathbf{u}_k) = 1$
  - $\rho^*_1 \geq \rho^*_2 \geq \rho^*_3 \geq \ldots \rho^*_{\min(m,n)} \geq 0$
  - **For** $j > k$, $\mathbf{u}_j$ **and** $\mathbf{w}_k$ **are uncorrelated, i.e.,**

$$\mathbf{u}_j'(\mathbf{I}-\mathbf{1}(\mathbf{1}'\mathbf{1})^{-1}\mathbf{1}')\,\mathbf{w}_k = 0$$

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… CanCorr (3)

- How does this work? From the zero scenario means, unit row norms, and definition of variance we get

$$\underset{(2m\times 1)}{\mathbf{0}} = \begin{bmatrix} \mathbf{T} \\ \mathbf{S} \end{bmatrix} \underset{(n\times 1)}{\mathbf{1}}$$

$$\underset{(2m\times 2m)}{\mathbf{\Sigma}} = \frac{1}{n - n_{zsm}} \begin{bmatrix} \mathbf{T} \\ \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{S} \end{bmatrix}'$$

$$= \begin{pmatrix} \mathbf{\Sigma}_{\mathbf{TT}} & \mathbf{\Sigma}_{\mathbf{TS}} \\ \mathbf{\Sigma}_{\mathbf{ST}} & \mathbf{\Sigma}_{\mathbf{SS}} \end{pmatrix} = \begin{pmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} \end{pmatrix}$$

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... CanCorr (4)

- Then it can be shown in about 3 textbook pages that

$$\begin{bmatrix} \mathbf{u}_k \\ \mathbf{w}_k \end{bmatrix} = \begin{bmatrix} \underbrace{\mathbf{e}_k{}'\mathbf{\Sigma}_{11}^{-1/2}}_{\mathbf{lca}_k}\mathbf{T} \\ \underbrace{\mathbf{f}_k{}'\mathbf{\Sigma}_{22}^{-1/2}}_{\mathbf{lcb}_k}\mathbf{S} \end{bmatrix}, \text{ where}$$

$$\text{Corr}(\mathbf{u}_k, \mathbf{w}_k) = \rho_k^* \text{ is the } k^{\text{th}} \text{ largest eigenvalue of}$$

$$\boxed{\mathbf{\Sigma}_{11}^{-1/2}\mathbf{\Sigma}_{12}\mathbf{\Sigma}_{22}^{-1}\mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1/2}}, \text{ with eigenvector } \mathbf{e}_k, \text{ and also of}$$

$$\boxed{\mathbf{\Sigma}_{22}^{-1/2}\mathbf{\Sigma}_{21}\mathbf{\Sigma}_{11}^{-1}\mathbf{\Sigma}_{12}\mathbf{\Sigma}_{22}^{-1/2}}, \text{ with eigenvector } \mathbf{f}_k.$$

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… CanCorr (5)

- First approach will be to look at initial canonical variables (cv's) as long as ρ_k are well separated
  - If ρ_k are closely spaced, eigenvectors and hence cv's are determined only up to rotations
- Second approach will be to look at residuals once projections on all cv's from 1 to k are removed, for k's such that
  - k is not in the middle of a closely spaced block of ρ_k
  - k is not so far beyond 1 that any patterns apparent in residuals are as likely to be artifacts of numerical methods as of remaining THUNDER vs. STORM relationships
- **We have not done this before; any advice would be welcome.**

SVD versus CanCorr: SVD (4)

- **A'A = Z of the SVD development can now be seen as Σ_{22} of CanCorr.**
- **We also have Σ_{12}, the covariation of T with S. We can estimate what we would expect for T based on S using the first k singular vectors of S as**

$$\hat{\mathbf{T}}(\mathbf{S},k) = \mathbf{\Sigma}_{12}\left(\mathbf{\Sigma}_{22}\right)^{-1|k} = \mathbf{\Sigma}_{12}\mathbf{O}\begin{pmatrix} d_{11}^{-1} & 0 & \cdots & & & \\ 0 & \ddots & & & & \\ \vdots & & d_{kk}^{-1} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}\mathbf{O}'$$

SVD versus CanCorr: SVD (5)

- Similar remarks and approaches as for CanCorr apply to SVD
  - Singular vectors are less well defined as eigenvalues become closely spaced
  - We may look at the fits (estimated values) first if most of the significance falls into a few numerically stable patterns
  - We will look at residuals as well, particularly at breaks in a block-type pattern of decrease in singular values / eigenvalues

SVD versus CanCorr: SVD (6)

- **The big, big difference between SVD and CanCorr is**
  - **SVD aims to compactly express the variation of a *single* data set**
  - **Covariation with the other data set is dealt with almost as an afterthought.**
- **CanCorr aims to compactly express the covariation of *both* data sets, without favor for one or the other.**

Automating Runs and Results (1)

- Again, a work in progress – all feedback welcome
- Number of parameters, factors, runs makes manual DOE setup and execution unattractive for all but small problems
  - Manual "baby steps" on small problems for debugging
- Concept is to factor problem into DOE files giving
  - Design matrix, source citation, permuted computational order
  - Factors and associated parameters, with .DAT file levels for parameters as .DOE factor goes between its levels

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Automating Runs and Results (2)

- Automate run process
  - Substituting parameter .DAT levels into .DOE files
  - Write result to temporary .DAT files
  - Execute runs in permuted order
- Automate DOE post process
  - Extract results
  - Transfer to statistical software
  - Use stat tools for evaluation and comparison between THUNDER, STORM results on nominally equivalent models

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Punch Line: Quantitative V&V?

- This is a massive quantitative validation experiment
  - Uses computers to exceed past human limits on data production
  - Uses computers and sophisticated math to surpass unaided human limits on data interpretation
  - Unexpected differences between THUNDER and STORM will constitute new knowledge
  - Finding only differences explainable as due to known deliberate changes would be a disappointment

Nothing to Lose But Uncertainty

- The author's involvement stems from > 20 years trying to compare test data with modeling results
  - 16 years shock testing nuclear powered ships and their components
  - 9 years with robotic test equipment testing composite material systems
    - Assumed material response theory (dissipated energy density) said could account for 60K data points via 125 coefficients and constraints
    - Ability to so compactly model test data gave massive support to underlying theory
- It is long past time for a similar quantitative approach to V&V – and comparing THUNDER and STORM should be a milestone in that effort.

Questions?